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On a Two-Dissimilar Unit Standby-Redundant System With Repair Under Economic Maintenance Policy

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Purpose : Report of a derivation
Special Math. needed: Probability, Laplace transforms
Results Useful to : Theoretically inclined reliability engineers.

Abstract- This paper discusses the case of a two-dissimilar units redundant repairable system under economic preventive maintenance policy, in which the preventive maintenance performs only if there exist a reserve element in standby ; if there is no element in standby the preventive maintenance does not perform and the operative elements are operating usually up to the failure moment of one of them.

We shall prove that the reliability function of the system tends to exponential one under the condition that the mean life time of the system is much larger than the mean repair time and the maintenance time. Also we study in this paper the effect of the repairable distribution functions on the mean life time of the system .

I. INTRODUCTION

The general standby redundant system with preventive maintenance, has been studied by several authors [1],[2], and [3] . The limiting distribution function of the duplex system of similar units with preventive maintenance for Moving, Economic, and Difficult policies is studied by [4] and [5] .

In case of two dissimilar units, the limiting distribution for time failure under Moving Policy is studied by [6] .

In this paper, the limiting distribution function for time failure of two dissimilar units standby redundant system with preventive maintenance under economic policy is studied in Sec (IV) . The effect of repairable distribution function on the mean life time of the system under economic policy is investigated in the last section (V) .

II- ASSUMPTIONS AND DEFINITIONS

1- The failure time distribution functions for the two units are $F_i(x)$, $i = 1,2$.

2- The replacement interval to inspection moment is a random variable with distribution functions $U_i(x)$, $i = 1,2$.

3- The repair and maintenance times are random variables with distributions $G_i(x)$, $V_i(x)$; and $V_i(x) > G_i(x)$, $i = 1,2$,

4- The time required for a unit to replace another one connected to repair or maintenance is negligible .

5- After repair and preventive maintenance each unit recovers its function perfectly and takes its place in standby immediately, if not required to operate.

6. All random variables are mutually independent and non-negative .

7- $Z(t)$ - the state variable of the system at instant t.

$Z(t) = 0$: The 1st unit operates and the 2nd is in standby .

$Z(t) = 1$: The 1st unit is under repair and the 2 nd unit operates.

$Z(t) = 2$: The 1st unit is under preventive maintenance and the 2nd unit operates .

$Z(t) = 3$: The 2nd unit is under repair and the 1st unit operates.

$Z(t) = 4$: The 2nd unit is under preventive maintenance and the 1st unit operates .

8- Φ_r the probability that the system is in state r, ($r = \overline{1,4}$) at the instant t=0 will work smoothly to the instant t.

9- R(t) the probability that the system is in state 0 at instant t=0 , will work smoothly to the instant t.

III - FORMULATION OF EQUATIONS

According to the above assumptions, R(t) and $\Phi_r(t)$, ($r = \overline{1,4}$) satisfying the following probabilistic integral equations.

$$R(t) = \overline{F}_1(t)\,\overline{U}_1(t) + \int_0^t \overline{U}_1(x)\,\Phi_1(t-x)\,dF_1(x) + \int_0^t \overline{F}_1(x)\,\Phi_2(t-x)\,dU_1(x) \quad \ldots \quad (1)$$

$$\Phi_1(t) = \overline{F}_2(t)\,\overline{G_1(t)U_2}(t) + \overline{F}_2(t)\int_0^t U_2(x)\,d\,G_1(x) + \int_0^t \overline{F}_2(x)\,G_1(x)\Phi_4(t-x)\cdot$$
$$dU_2(x) + \int_0^t \overline{U}_2(x)\,G_1(x)\,\Phi_3(t-x)\,d\,F_2(x) + \int_0^t \Phi_3(t-x)\cdot$$
$$\int_0^x U_2(Y)\,dG_1(y)\,dF_2(x) \quad \ldots\ldots\ldots \quad (2)$$

$$\Phi_2(t) = \overline{F}_2(t)\,\overline{V_1(t)\;U_2}(t) + \overline{F}_2(t)\int_0^t U_2(x)\,dV_1(x) + \int_0^t \overline{F}_2(x)\,V_1(x)\,\Phi_4(t-x)\cdot$$
$$dU_2(x) + \int_0^t \overline{U}_2(x)V_1(x)\,\Phi_3(t-x)\,dF_2(x) + \int_0^t \Phi_3(t-x)\int_0^x U_2(y)$$
$$dV_1(y)\,dF_2(x) \quad \ldots\ldots \quad (3)$$

$$\Phi_3(t) = \overline{F}_1(t)\,\overline{G_2(t)U_1}(t) + \overline{F}_1(t)\int_0^t U_1(x)\,dG_2(x) + \int_0^t \overline{F}_1(x)\,G_2(x)\,\Phi_2(t-x).$$

$$dU_1(x) + \int_0^t \overline{U}_1(x)G_2(x)\,\Phi_1(t-x)\,dF_1(x) + \int_0^t \Phi_1(t-x)\int_0^x U_1(y)\,dG_2(y).$$

$$dF_1\ (x) \quad \ldots\ldots \tag{4}$$

$$\Phi_4(t) = \overline{F}_1(t)\,\overline{V_2(t)U_1}(t) + \overline{F}_1(t)\int_0^t U_1(x)\,dV_2(x) + \int_0^t \overline{F}_1(x)\,V_2(x)\,\Phi_2(t-x)\,dU_1(x) +$$

$$+\int_0^t \overline{U}_1(x)V_2(x)\,\Phi_1(t-x)\,dF_1(x) + \int_0^t \Phi_1(t-x)\int_0^x U_1(y)\,dV_2(y)\,dF_1(x)\ \ldots \tag{5}$$

Taking Laplace-Stieltjes(LS) transform for equations (1-5), and using the following notations :

$$\alpha_{11}(s) = \int_0^\infty e^{-st}\,\overline{F}_1(t)\,dU_1(t) \quad ; \quad \alpha_{21}(s) = \int_0^\infty e^{-st}\,\overline{U}_1(t)\,dF_1(t)$$

$$f_1(s) = \int_0^\infty e^{-st}\,dF_1(t) \quad ; \quad f_2(s) = \int_0^\infty e^{-st}\,dF_2(t)$$

$$g_1(s) = \int_0^\infty e^{-st}\,G_1(t)\,dF_2(t) \quad ; \quad \tilde{g}_1(s) = \int_0^\infty e^{-st}\,G_2(t)\,dF_1(t)$$

$$g_2(s) = \int_0^\infty e^{-st}V_1(t)\,dF_2(t) \quad ; \quad \tilde{g}_2(s) = \int_0^\infty e^{-st}\,V_2(t)\,dF_1(t)$$

$$c_{11}(s) = \int_0^\infty e^{-st}\,\overline{U}_2(t)\,G_1(t)\,dF_2(t) \ ; \quad c_{12} = \int_0^\infty e^{-st}\,\overline{U}_1(t)\,G_2(t)\,dF_1(t)$$

$$c_{21} = \int_0^\infty e^{-st}\,\overline{U}_1(t)\,V_2(t)\,dF_1(t) \ ; \quad c_{22}(s) = \int_0^\infty e^{-st}\,\overline{U}_2(t)\,V_1(t)\,dF_2(t)$$

$$d_{11}(s) = \int_0^\infty e^{-st}\,\overline{F}_2(t)G_1(t)\,dU_2(t) \ ; \quad d_{12} = \int_0^\infty e^{-st}\,\overline{F}_1(t)\,G_2(t)\,dU_1(t)$$

$$d_{21} = \int_0^\infty e^{-st}\, \overline{F}_1(t) V_2(t)\, dU_1(t) \quad ; \quad d_{22}(s) = \int_0^\infty e^{-st}\, \overline{F}_2(t)\, V_1(t) dU_2(t)$$

$$e_{12}(s) = \int_0^\infty e^{-st} \left(\int_0^t U_2(y) dG_1(y) \right) dF_2(t) \quad ; \quad \tilde{e}_{12}(s) = \int_0^\infty e^{-st} \left(\int_0^t U_2(y) dV_1(y) \right) dF_2(t)$$

$$e_{21}(s) = \int_0^\infty e^{-st} \left(\int_0^t U_1(y) dG_2(y) \right) dF_1(t) \quad ; \quad \tilde{e}_{21}(s) = \int_0^\infty e^{-st} \left(\int_0^t U_1(y) dV_2(y) \right) dF_1(t)$$

$$\overline{F}_i(t) = 1 - F_i(t) ; \ \overline{G}_i(t) = 1 - G_i(t) \ ; \ \overline{V}_i(t) = 1 - V_i(t) \ ; \ \overline{U}_i(t) = 1 - U_i(t) \qquad \ldots\ldots \quad (6)$$

We obtain

$$R(s) = - \int_0^\infty e^{-st}\, dR(t) = \frac{I_6[\alpha_{11} I_1 + \alpha_{21} I_2] + I_5[\alpha_{11} I_3 + \alpha_{21} I_4]}{I_1 I_4 - I_2 I_3} \qquad \ldots\ldots \quad (7)$$

Where

$$I_1 = 1 - d_{11}(c_{21} + \tilde{e}_{21}) - (c_{11} + e_{12})(c_{12} + e_{21}) \ ; \quad I_2 = d_{11} d_{21} + d_{12}(c_{11} + e_{12})$$

$$I_3 = d_{22}(c_{21} + \tilde{e}_{21}) + (c_{22} + \tilde{e}_{12})(c_{12} + e_{21}) \ ; \quad I_4 = 1 - d_{22} d_{21} - d_{12}(c_{22} + \tilde{e}_{12})$$

$$I_5 = f_2 - g_1 + d_{11}(f_1 - \tilde{g}_2) + (c_{11} + e_{12})(f_1 - \tilde{g}_1)$$

$$I_6 = f_2 - g_2 + d_{22}(f_1 - \tilde{g}_2) + (c_{22} + \tilde{e}_{12})(f_1 - \tilde{g}_1)$$

The mean time to the system failure (MTSF) $= - \left. \dfrac{dR(s)}{ds} \right|_{s=0}$ ..... (8)

IV- THE LIMITING FAILURE TIME DISTRIBUTION OF THE SYSTEM.

We shall prove in this section that in case of a constant time preventive maintenance policy,

$$U_i(t) = \begin{cases} 1, & \text{for } t \geq T \\ 0, & \text{for } t < T \end{cases} \quad , \quad (i = 1,2) \ ,$$

and under the condition that the time of repair T_R and the time of inspection T_I are more smaller than the operating time T_o of the main equipment, the destribution function of the system will tends to an exponential one .

Proof:- Since $T_o >> T_R$; $T_o >> T_I$, then there exist y, ε_{yi} , $(i=\overline{1,4})$ such that

$\overline{G}_{iy}(t) < \varepsilon_{yi}$; $\overline{V}_{iy}(t) < \varepsilon_{yj}$, (i=1,2, j = 3,4) , t >

Thus,

$$\lim_{y\to\infty} \overline{G}_{iy}(t) = 0 \quad ; \quad \lim_{y\to\infty} \overline{V}_{iy}(t) = 0 \quad ,(i=1,2) \; ,$$

$$\left.\begin{array}{l} \delta_1 = \int_0^\infty \overline{G}_{1y}(t)\, dF_2(t) \xrightarrow[y\to\infty]{} 0 \quad ; \quad \delta_2 = \int_0^\infty \overline{V}_{1y}(t)\, dF_2(t) \xrightarrow[y\to\infty]{} 0 \\ \delta_3 = \int_0^\infty \overline{G}_{2y}(t)\, dF_1(t) \xrightarrow[y\to\infty]{} 0 \; ; \quad \delta_4 = \int_0^\infty \overline{V}_{2y}(t)\, dF_1(t) \xrightarrow[y\to\infty]{} 0 \end{array}\right\} \qquad (9)$$

Choosing the parameter α_y ,

$$\alpha_y = F_1(T)\,\delta_1 + \overline{F}_1(T)\,\delta_2 + \overline{F}_2(T)\,\delta_3 + F_2(T)\,\delta_4 \qquad (10)$$

Then from (6),(7),(9) and (10) , the Laplace transform of the distribution function of the operating time of the system given as a function of $\alpha_y s$ has the form

$$R(\alpha_y s) = \frac{L_4 + L_5 + O(\alpha_y^2)}{L_1 + L_2 + L_3 + O(\alpha_y^2)} \quad ,$$

where

$$L_1 = e^{-2\alpha_y sT}\,\overline{F}_1(T)\,\overline{F}_2(T)\,[V_{1y}(T) - G_{1y}(T)]\,[G_{2y}(T) - V_{2y}(T)]\,[1 - \int_0^\infty e^{-\alpha_y st} dF_2(t) \cdot \int_0^\infty e^{-\alpha_y st}\, dF_1(t)\,]$$

$$L_2 = 1 - \int_0^\infty e^{-\alpha_y st} dF_2(t) \int_0^\infty e^{-\alpha_y st} dF_1(t) - \alpha_y s \, [G_{1y}(T) \int_T^\infty e^{-\alpha_y st} \bar{F}_2(t) dt .$$

$$\int_0^\infty e^{-\alpha_y st} dF_1(t) + G_{2y}(T) \int_T^\infty e^{-\alpha_y st} \bar{F}_1(t) dt \int_0^\infty e^{-\alpha_y st} dF_2(t) \,]$$

$$L_3 = \sum_{\substack{i,j=1 \\ i \neq j}}^{2} \Big\{ e^{-\alpha_y sT} \bar{F}_i(T) \, G_{iy}(T) \int_0^\infty e^{-\alpha_y st} \bar{V}_{iy}(t) \, dF_j(t) + \int_0^\infty e^{-\alpha_y st} dF_i(t) .$$

$$\int_0^\infty e^{-\alpha_y st} \bar{G}_{iy}(t) dF_j(t) - G_{iy}(T) \int_T^\infty e^{-\alpha_y st} dF_i \int_0^\infty e^{-\alpha_y st} \bar{G}_{iy}(t) dF_j(t) \Big\}$$

$$L_4 = \int_0^T e^{-\alpha_y st} \, dF_1(t) \int_0^\infty e^{-\alpha_y st} \bar{G}_{1y}(t) \, dF_2(t) +$$

$$e^{-\alpha_y sT} \, \bar{F}_1(T) \int_0^\infty e^{-\alpha_y st} \bar{V}_{1y}(t) \, dF_2(t) .$$

$$L_5 = \Big\{ \int_0^T e^{-\alpha_y st} \, dF_1(t) + e^{-\alpha_y sT} \bar{F}_1(T) \Big\} \int_0^\infty e^{-\alpha_y st} \bar{G}_{2y}(t) \, dF_1(t) \int_0^\infty e^{-\alpha_y st} dF_2(t)$$

$$+ \Big\{ e^{-\alpha_y sT} \bar{F}_2(T) G_{1y}(T) \int_0^T e^{-\alpha_y st} dF_1(t) + e^{-2\alpha_y sT} \bar{F}_1(T) \bar{F}_2(T) V_{1y}(T) \Big\} .$$

$$. \int_0^\infty e^{-\alpha_y st} \bar{V}_{2y}(t) \, dF_1(t)$$

$$- \Big\{ G_{1y}(T) \int_0^T e^{-\alpha_y st} \, dF_1(t) + e^{-\alpha_y sT} \bar{F}_1(T) V_{1y}(T) \Big\} \int_T^\infty e^{-\alpha_y st} dF_2(t) .$$

$$\int_0^\infty e^{-\alpha_y st} \, \bar{G}_{2y}(t) \, dF_1(t) \qquad \ldots . \qquad (1$$

From the above assumptions,(9) and (10), we have :

$$L_1 / \alpha_y \xrightarrow[\alpha_y \to 0]{} 0 \; ; \; L_2/\alpha_y \xrightarrow[\alpha_y \to 0]{} s(T_1+T_2) \; ; \; L_3/\alpha_y \xrightarrow[\alpha_y \to 0]{} 1 \ldots \quad (12)$$

$$[L_4 + L_5] / \alpha_y \xrightarrow[\alpha_y \to 0]{} 1 \; , \ldots \quad (13)$$

where $$T_1 = \int_0^T \overline{F}_1(t)\, dt \; ; \quad T_2 = \int_0^T \overline{F}_2(t)\, dt \quad (14)$$

Therefore from (12),(13) and (14) we obtain

$$R(\alpha_y, s) \xrightarrow[\alpha_y \to 0]{} \frac{1}{1 + s(T_1 + T_2)}$$

Which means that the distribution function of the system tends to exponential one .

V. THE EFFECT OF REPAIR DISTRIBUTIONS ON THE MEAN LIFE TIME IN CASE OF ECONOMIC MAINTENANCE POLICY

In this section, we study through anumerical example the effect of repair distribution functions on the mean life time of the system T_m. We consider here an age preventive maintenance policy which could be applied in practical fields

that is , we assume

$$U_i(t) = \begin{cases} 1, & \text{for } t \geq T \\ 0, & \text{for } t < T \end{cases} \quad , \quad (i= 1,2) \quad (15)$$

Using (6),(7),(8) and (15), we have

$$T_m = \int_0^T \overline{F}_1(t)\, dt + \frac{[I_4 I_5 + I_2 I_6]\, F_1(T) + [I_1 I_6 + I_3 I_5]\, \overline{F}_1(T)}{(I_1 I_4 - I_2 I_3)} \quad (16)$$

$$I_1 = 1 - [\int_0^\infty G_2(t)dF_1(t) - G_2(T)\,\overline{F}_1(T)]\,[\int_0^\infty G_1(t)dF_2(t) - G_1(T)\,\overline{F}_2(T)]$$

$$- \overline{F}_2(T)\,G_1(T)\,[\int_0^\infty V_2(t)\,dF_1(t) - V_2(T)\,\overline{F}_1(T)]$$

$$I_2 = \overline{F}_1(T)\,G_2(T)\,[\int_0^\infty G_1(t)dF_2(t) - G_1(T)\,\overline{F}_2(T)] + \overline{F}_1(T)\overline{F}_2(T)\,G_1(T)V_2(T)$$

$$I_3 = [\int_0^\infty G_2(t)dF_1(t) - G_2(T)\overline{F}_1(T)]\,[\int_0^\infty V_1(t)dF_2(t) - V_1(T)\,\overline{F}_2(T)] + \overline{F}_2(T)V_1(T)$$

$$[\int_0^\infty V_2(t)\,dF_1(t) - V_2(T)\,\overline{F}_1(T)]$$

$$I_4 = 1 - \overline{F}_1(T)\,G_2(T)\,[\int_0^\infty V_1(t)\,dF_2(t) - V_1(T)\overline{F}_2(T)] - \overline{F}_1(T)\overline{F}_2(T)\,V_1(T)\,V_2(T)$$

$$I_5 = \int_0^\infty \overline{F}_2(t)\,dt - G_1(T)\int_T^\infty \overline{F}_2(t)\,dt + \overline{F}_2(t)\,G_1(T)\,[\int_0^\infty \overline{F}_1(t)dt - V_2(T)\int_T^\infty \overline{F}_1(t)dt]$$

$$+ [\int_0^\infty \overline{F}_1(t)\,dt - G_2(T)\int_T^\infty \overline{F}_1(t)\,dt]\,[\int_0^\infty G_1(t)\,dF_2(t) - G_1(T)\,\overline{F}_2(T)]$$

$$I_6 = \int_0^\infty \overline{F}_2(t)\,dt - V_1(T)\int_T^\infty \overline{F}_2(t)\,dt + \overline{F}_2(T)\,V_1(T)\,[\int_0^\infty \overline{F}_1(t)\,dt - V_2(T)\int_T^\infty \overline{F}_1(t)dt]$$

$$+ [\int_0^\infty \overline{F}_1(t)\,dt - G_2(T)\int_T^\infty \overline{F}_1(t)dt]\,[\int_0^\infty V_1(t)\,dF_2(t) - V_1(T)\,\overline{F}_2(T)]$$

Let

$F_i(t) = 1-(1+\lambda_i t)\,e^{-\lambda_i t}$; $V_i(t) = 1 - e^{-\gamma_i t}$; $i = 1,2$, where

$\lambda_1 = 0.01$; $\lambda_2 = 0.02$; $\gamma_1 = 0.333$ and $\gamma_2 = 0.667$, the mean life time T_m given by (16) is calculated as a function of T for the following repair time distributions :

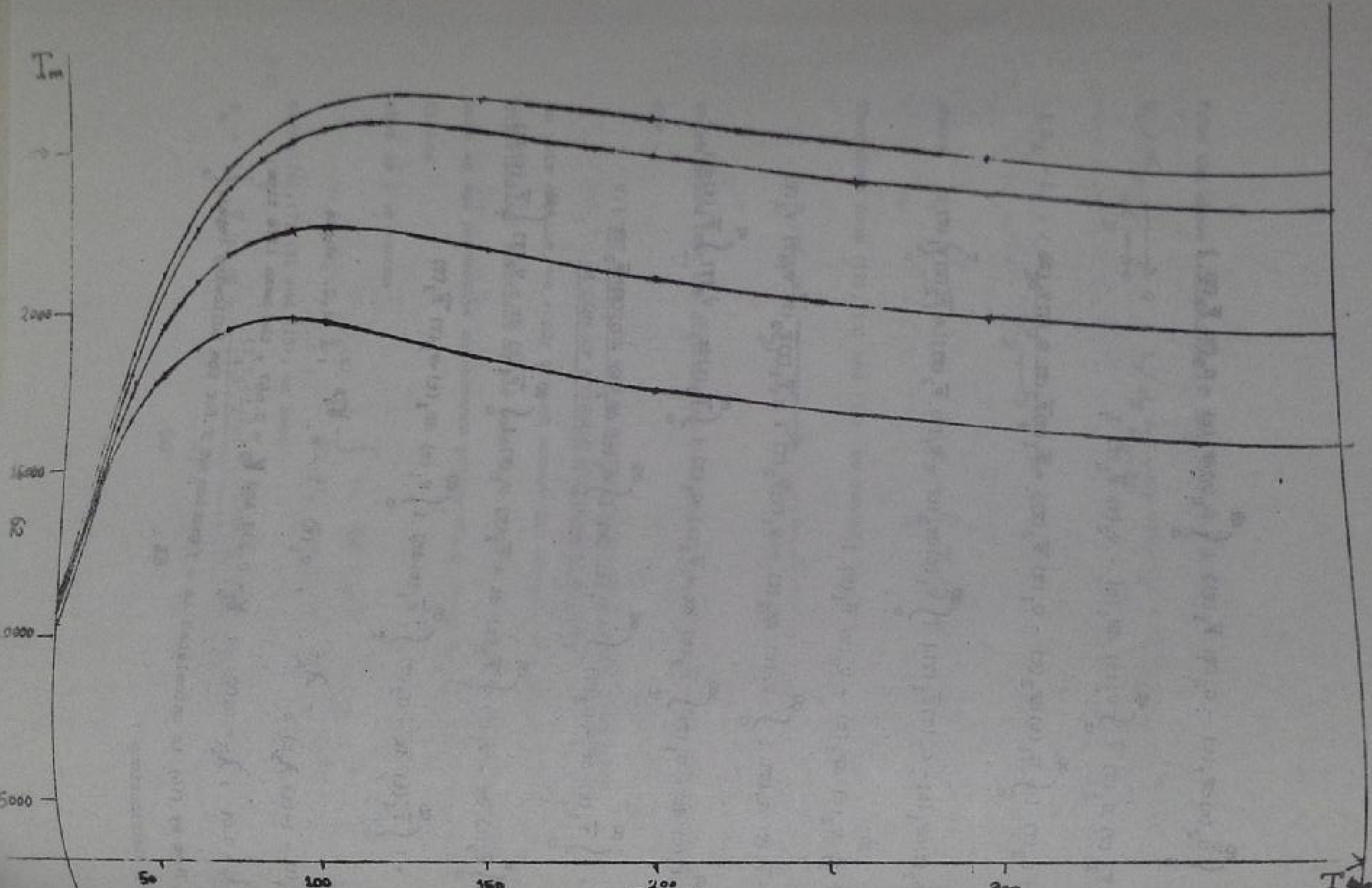
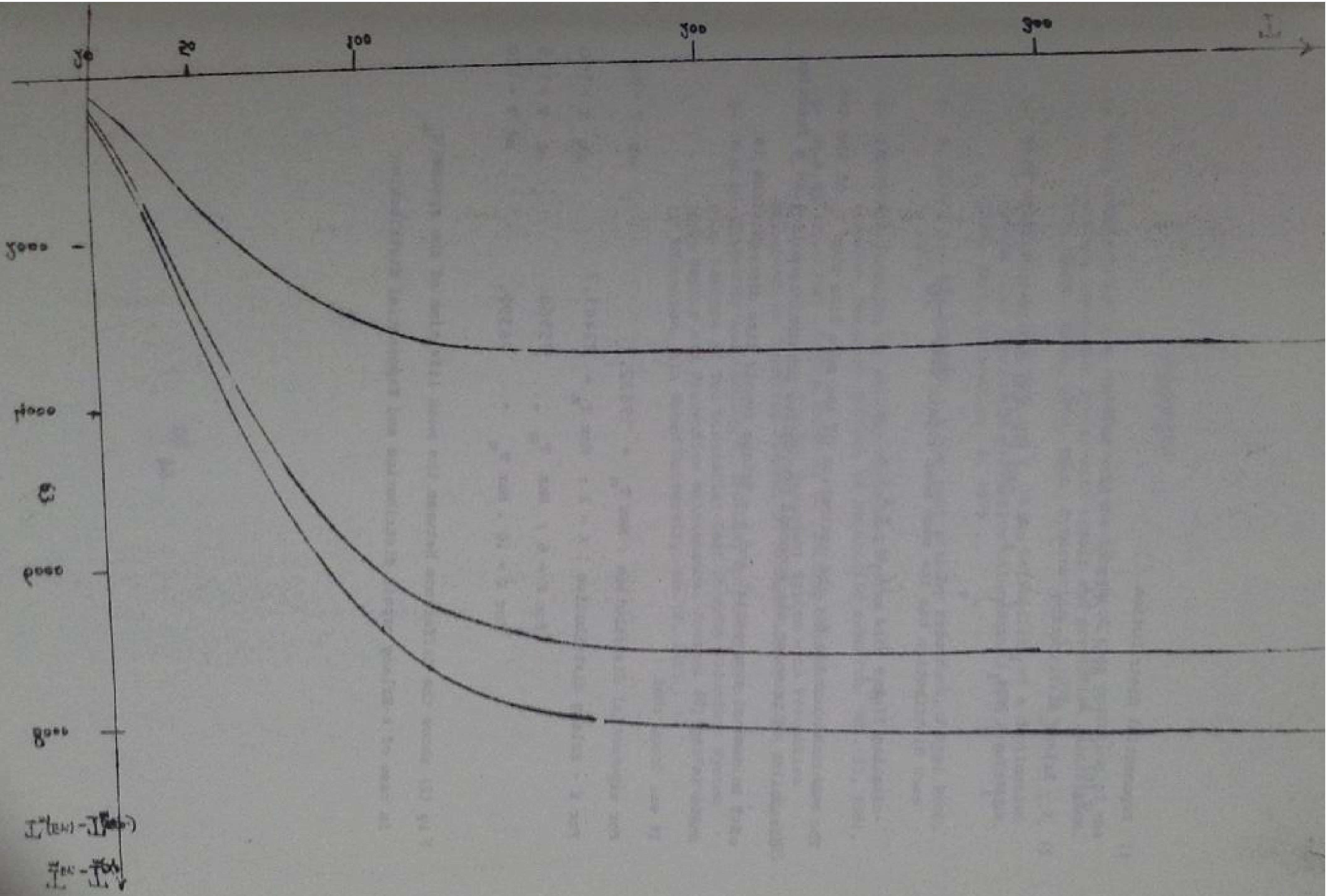


Fig (1)



1) Exponential distribution

$$G_i(t) = 1 - \exp(-\mu_i t) , \qquad (i = 1,2)$$

2) K - Erlang distribution

$$G_i(t) = (k\mu_i)^k \int_0^t t^{k-1} \exp(-k\mu_i t)\, dt/ (k-1)!, \quad (i = 1,2)$$

each distribution has the same mean repair time $\frac{1}{\mu_i}$ -

(taking $1/\mu_1 = 8$; $1/\mu_2 = 4$) .

The numerical results for the estimation of the mean life time T_m of the two dissimilar units under the economic preventive maintenance policy as a functi[illegible] of T in case of exponential and k - Erlang repair time distributions is shown in fig (1) .

It was found that :

For exponential distribution ; Max T_m = 19622.7 at $T =$ [illegible]

For K - Erlang distribution , K = 2 ; Max T_m = 22441.7 at $T =$ [illegible]

for K = 6 ; Max T_m = 25560. at $T =$ [illegible]

for K = 10 , Max T_m = 26392. at $T =$ [illegible]

Fig (2) shows the difference between the mean life time of the system T_m in case of k-Erlang repair distribution and Exponential distribution .

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دراسة نظام ثنائى غير متماثل تحت تاثير الصيانة الاقتصادية

ملخص البحث

تناول هذا البحث دراسة نظام ثنائى غير متماثل تحت تاثير الصيانة الاقتصادية. بفرض ان عملية الصيانة الوقائية سوف تتم فقط اذا وحدت الة احتياطية، وان كان الاحتياطى فى التصليح او الصيانة فان الوحدة العاملة تستمر فى العمل حتى تتوقف

ولقد اثبتنا نظريا ان نهاية التوزيع الاحتمالى لصلاحية النظام عندما يكون الزمن المتنقل على التصليح والزمن المتفق على الصيانة اقل بكثير من عمر الالة يؤل الى التوزيع الاسى .

وقد بينا خلال مثال عددى تاثير نوع دالة التصليح على عمر النظام ووجد ان عمر النظام فى حالة دالة التصليح هى توزيع ايرلنج اكبر بكثير عنه فى حالة دالة التصليح توزيع اسي . كما ان عمر النظام والنهاية العظمى له فى حالة دالة التصليح توزيع ايرلنج يتوقف على قيمة البارمتر k الداخل فى دالة التوزيع فانه يزداد بازدياده .

المؤتمر الدولى الثانى عشر

للاحصاء والحسابات العلمية والبحوث الاجتماعية والسكانية

القاهرة ـ جمهورية مصر العربية

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